FORMULATION OF THE INVERSE COEFFICIENT PROBLEMS OF HYDROGEOLOGY

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A new formulation of inverse coefficient problems that are unidentified as a whole is proposed.

The solution of inverse coefficient problems from observational data is an urgent topic in hydrogeological research [1]. In [2], regions of possible values of the coefficients were found. There arises the question of the mutually unique correspondence between the coefficients of the equation and its solutions. In [3], some work was done with regard to this question for the heat-conduction equation. If this mutually unique correspondence is established, it may form the basis for the formulation of inverse coefficient problems that are unidentified as a whole. At present, the formulation of inverse coefficient problems with respect to observational data both for hydrogeological equations and for heat-conduction equations [4] is based on the minimization of the functional

$$J = \sum_{i=1}^{N} (U_i^{\delta} - U_i)^2. \tag{1}$$

It is a function of the coefficients of the equation and is calculated from the U-model solution of the usual direct problem with these (desired) coefficients.

The following initial hydrogeological problem is used for the nonsteady filtration equation in the case of a single experimental borehole operating at constant output Q from a water-bearing level bounded by a guaranteed-supply loop with a constant head H_0:

$$\mu H_t = \nabla (T \nabla H), \tag{2}$$

$$H|_{t=0} = \psi,\ T\frac{\partial H}{\partial n}|_{r=\rho} = Q,\ H|_{r=R} = H_0, \tag{3}$$

$0 < \rho << R$; μ, T, ψ are functions of (r, φ) in the polar coordinate system.

By analogy with [3], it is assumed that the function U* is a solution of Eqs. (2), (3) with two different pairs of coefficients (μ_1, T_1), (μ_2, T_2). Then it is found that U* is the solution of the following problem:

$$\mu^* U_t^* = \nabla (T^* \nabla U^*), \tag{4}$$

$$U^*|_{t=0} = \psi,\ \frac{\partial U^*}{\partial n}\bigg|_{r=\rho} = 0,\ U^*|_{r=R} = H_0,\ \mu^* = \mu_1 - \mu_2,\ T^* = T_1 - T_2. \tag{5}$$

Introducing the linear space X = {(μ, T)} with specified Q, H_0, and ψ, the G set of solutions of Eqs. (2), (3) with (μ, T) $\in$ X is considered. Let K be the set of {U^*} solutions of Eqs. (4), (5) with (μ^*, T^*) $\in$ X and ψ and H_0 be proportional to the initial quantities.

Studying the factor-set G/K, it is noted that there is mutually unique correspondence between X and G/K.

Thus, there is an identification not between X and G, but between X and G/K, where the element G/K is a set of functions of the form $U^\alpha = H + \alpha U^*$; α is a real number. The function U^α is the solution of Eqs. (2), (6):

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$$U^{\alpha}|_{t=0} = \psi(1+\alpha),\ T\frac{\partial U^{\alpha}}{\partial n}\bigg|_{r=\rho} = Q,\ U^{\alpha}|_{r=R} = H_0(1+\alpha). \tag{6}$$

On the basis of this mutually unique correspondence, a new formulation of the solution of inverse coefficient problems with respect to observational data may be proposed. It is based not on the minimization of the functional in Eq. (1) but on the minimization of the functional

$$J_{\text{new}} = \sum_{i=1}^{N}(U_i^{\delta} - U_i^{\alpha})^2.$$

It is a function of α and the coefficients of the equation, and is calculated from the U^{α} solution of the problem in Eqs. (2), (6).

Thus, in the proposed formulation of the inverse problems there appears an additional independent variable α, with respect to which the minimum of the functional must also be sought, and U^{α} (the model solution) will also be the solution of another direct problem.

NOTATION

N, number of observation points; U_i^{δ}, observational data; U, model solution of the direct problem; H, head; ψ, initial value of the head; r, φ , coordinates in the polar coordinate system; Q, output of borehole; μ, water yield; H_0, head at the guaranteed-supply loop; T, water conduction; ρ, borehole radius; R, radius of guaranteed-supply loop; X, linear space of coefficients of the equation; G, K, sets; G/K, factor-set; U^{α}, element of the factor-set; α, real number.

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